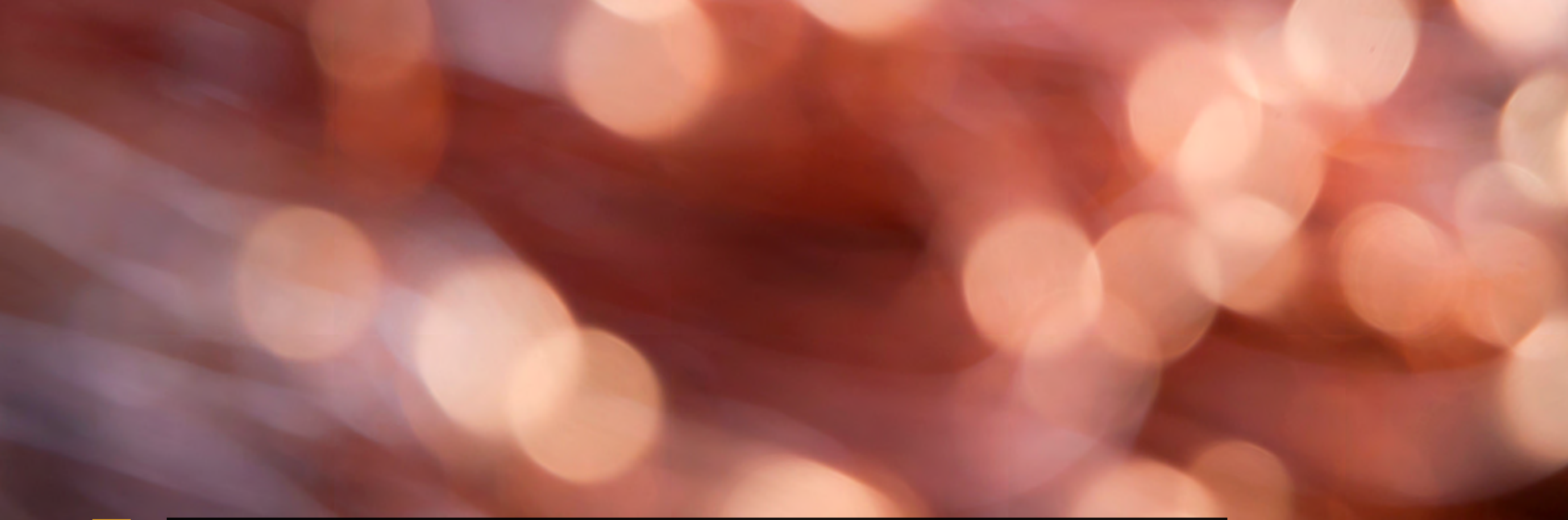
TRIAD MAGNETICS

An Introduction to Impedance Matching Transformers



INTRODUCTION

Many applications — audio applications, data networks, communications — require transmitting an AC signal from a source to a load while minimizing distortion and maximizing power transfer.

To ensure optimal performance, these systems rely on a process called impedance matching. In this paper, we'll take a look at why impedance matching is so important, how it works, and how it can be easily implemented during system design through proper impedance matching transformer selection.

What is Impedance?

Electrical impedance is a measure of the opposition to the flow of current within a circuit or electrical component. It is made up of two parts, **Resistance** and **Reactance**.

- **Resistance** is the opposition to the flow of current caused by the physical properties of the electrical components. The value is proportional to the voltage and current.
- **Reactance** is the opposition of an electrical component or circuit to a change in current or voltage. It can be broken down into capacitive reactance, which opposes any change in voltage, and inductive reactance, which opposes any change in current. Capacitive reactance is inversely proportional and inductive reactance is directly proportional to the frequency of applied voltage.

Impedance, which combines resistance and reactance, is a complex mathematical number, meaning it has both real and imaginary components. This is important when calculating impedance values.

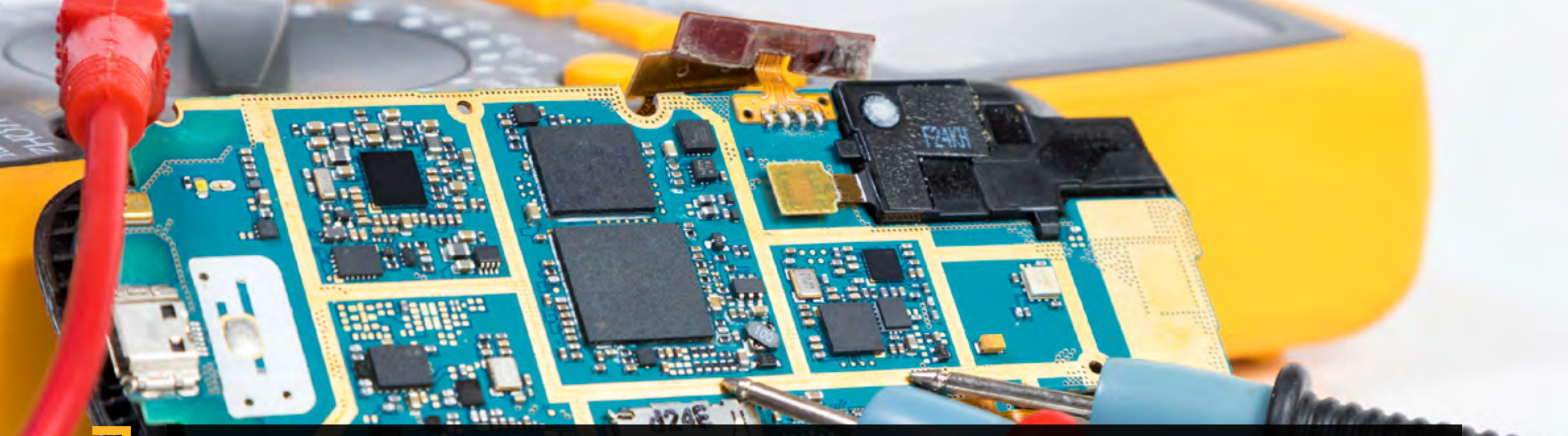
$$Z = R + jX$$

Z = Impedance **R** = Resistance (Real component)
X = Reactance (Imaginary component, designated by j)

DC circuits are a special case. In a DC circuit where frequency is 0, reactance of an inductor is 0 and the reactance of a capacitor is infinite.

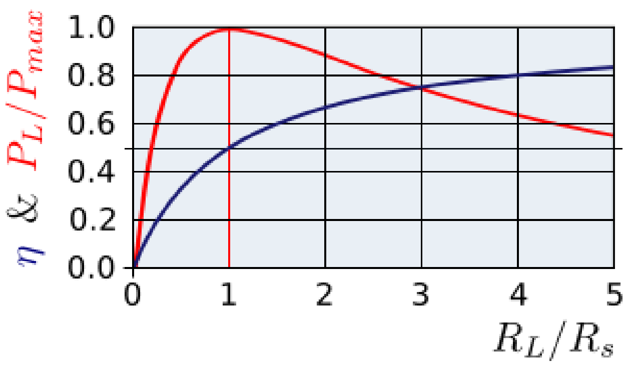
AC circuits include all electrical devices that are plugged into a wall socket at home or in a factory, as well as radio transmission systems, sonic delivery systems (including hi-fi and ultrasonic detection equipment), and the transmission of electrical power over long distances — most of the circuits you come into contact with on a day-to-day basis.

This paper will delve straight into how impedance affects your circuit. However, to know more about how these values are calculated, skip to the **Calculations and Proofs** section for a full explanation.



MAXIMUM POWER TRANSFER THEOREM: HOW IMPEDANCE AFFECTS POWER TRANSFER IN A CIRCUIT

The ratio of impedance in the source and the load affect how much power can be transferred from the source to the load. For maximum power transfer (assuming a fixed source impedance), the source impedance and load impedance must be equal in magnitude.



η = efficiency

P_L = load power

P_{max} = max power

R_L = load resistance

R_S = source resistance

For DC circuits, efficiency is essentially the power dissipated at the load as a fraction of power dissipated at the source. Therefore, efficiency is calculated through the following equation:

$$\eta = \frac{R_L}{R_L + R_S}$$

A plot of efficiency and power transfer ratio as a function of R_L/R_S is shown in figure above. It can be seen that maximum power transfer occurs when the source resistance and load resistance are equal. The power curve above shows that a maximum power transfer system has efficiency losses of 50%.

For AC circuits, it is the impedance that needs to be considered. If the impedance of the source is lower than the impedance of the load, the magnitude of power transfer is reduced, but the efficiency of the transfer is increased.

For maximum power transfer in AC circuits, the source impedance and load impedance must be complex conjugates — meaning the source resistance and load resistance must be the same, while the source reactance and load reactance must be equal in magnitude but opposite in phase.

$$Z_{source} = R + jX \qquad Z_{load} = R - jX$$

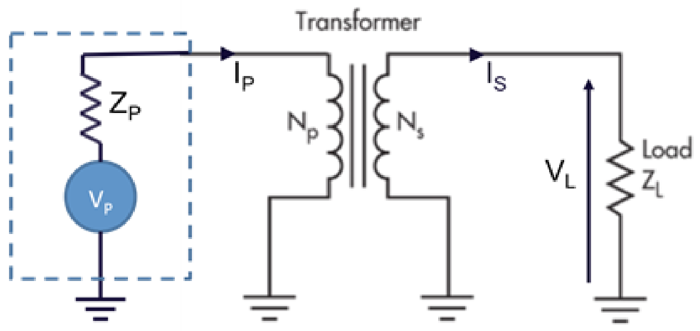
For most applications, maximum power transfer is chosen over maximum efficiency. The power that is not transferred is dissipated as heat.

IMPEDANCE MATCHING

It's clear that being able to match the load impedance is essential to the functionality of the circuit, especially for maximum power transfer. The simplest method of matching the load impedance is to use a transformer.

Transformers

A transformer is a highly efficient (between 95-98.5%) device used to transfer electrical energy from one circuit to another through electromagnetic induction. Transformers are made by wrapping two coils of wire (windings) around an iron/ferrite core. When an excitation voltage is applied to the primary winding, a magnetic field is developed in the core, which then induces a current in the *secondary winding*.



V_P= primary voltage
Z_P= primary impedance
I_P= primary current
N_P= primary no. of turns
V_L= /load voltage
Z_L= /load impedance
I_S= secondary current
N_S=secondary no. of turns

The ratio of primary voltage to secondary/load voltage is the same as the ratio of primary winding no. of turns to secondary winding no. of turns. This ratio will be called n.

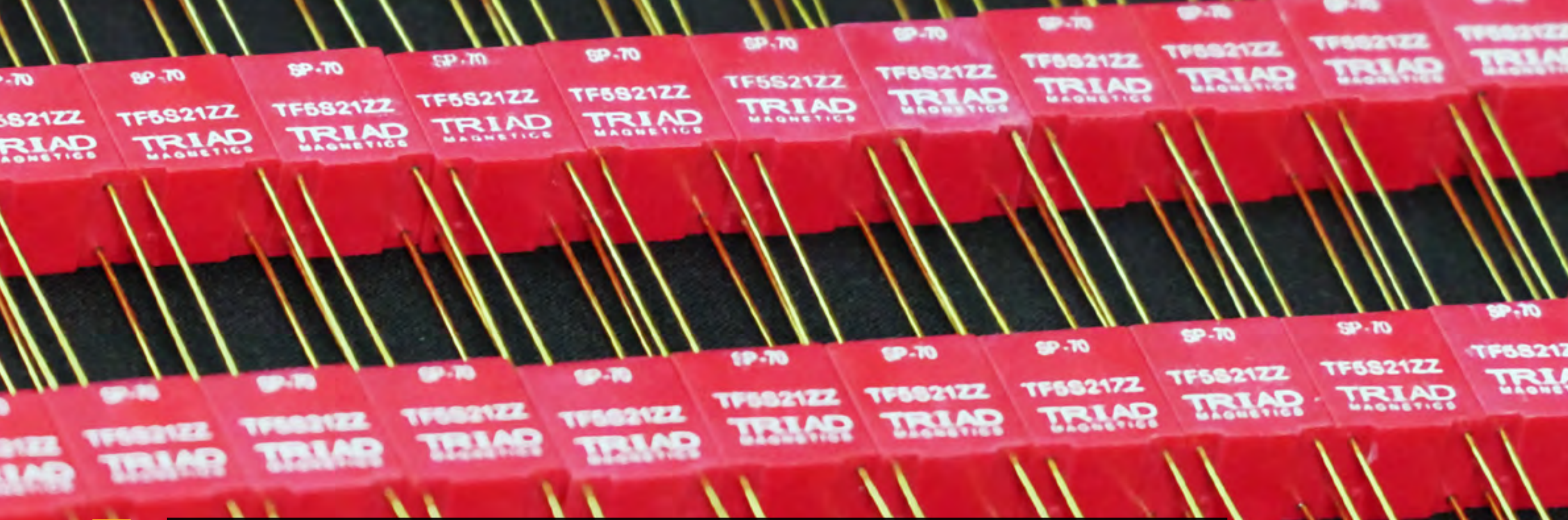
$$\frac{V_p}{V_L} = \frac{I_S}{I_P} = \frac{N_P}{N_S} = n$$

The primary/source impedance and secondary/load impedance are also related via n. This is often known as the **Impedance Ratio** of the transformer circuit.

$$n^2 = \frac{Z_p}{Z_S}$$

You can see how these factors are fully calculated in the **Calculations and Proofs** section.

This shows that the load impedance can be quite simply controlled by selecting the correct transformer specifications for the chosen circuit, allowing for maximum power transfer.



IMPEDANCE MATCHING IN PRACTICE

Problem: An audio amplifier is required to drive a load at maximum available power. The impedance of the amplifier is 300 Ω and the impedance of the load is 75 Ω. When connected directly, the signal transfer is poor.

Solution: The difference between source impedance and load impedance is causing the poor signal transfer. By using an impedance matching transformer, the load impedance can be matched to the source impedance.

What transformer winding ratio is required to match the load impedance to the source impedance?

$$n^2 = \frac{Z_p}{Z_s} = \frac{300}{75} = 4 \qquad n = 2$$

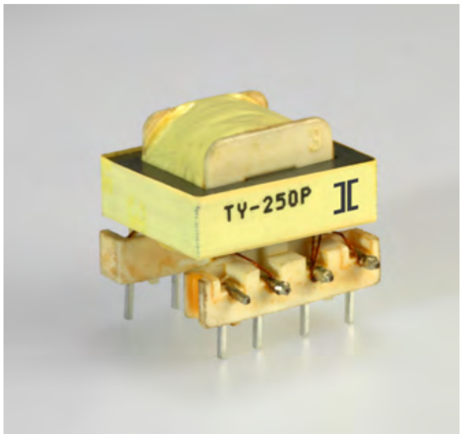
A 2:1 winding ratio is required.

Problem: The 75 Ω load has been swapped out for a 12 Ω load. What is the new transformer winding ratio required for maximum power transfer in this circuit?

Solution:

$$n^2 = \frac{Z_p}{Z_s} = \frac{300}{12} = 25 \qquad n = 5$$

A 5:1 winding ratio is required.



APPLICATIONS AND CONSIDERATIONS FOR IMPEDANCE MATCHING TRANSFORMERS

Impedance matching transformers can be used in any AC circuit, but are most frequently used for audio equipment, microphones, amplifiers, data networks/data systems, telephone grids, phone systems, and communication on airplanes — any application where power transfer must be maximized and distortion minimized.

While working out the impedance ratio of a circuit is simple, selecting a transformer in practice is a little more complex. Several factors must be taken into consideration, including:

Core Saturation — Magnetic materials (such as the core of a transformer) can only absorb up to a certain density of magnetic flux. When an electromagnetic device approaches magnetic flux saturation, it begins to behave in a highly nonlinear manner. The higher the signal level, the higher the transformer's magnetic flux.

When the core is saturated, the primary signal cannot be faithfully reproduced in the secondary side of the circuit; this results in signal clipping, where peak values of voltage and current are dropped, as the required power output is greater than what the transformer can provide.

Core saturation can be avoided by choosing a larger transformer made from the same materials or using a core material that saturates at a greater magnetic flux density. Lots of time and money are being spent researching exotic new materials and configurations for this very purpose! The maximum power capability determines the minimum size of the transformer.

Frequency — Due to similar nonlinear effects, transformers only perform adequately over a limited range of frequencies. While the transformers for mains power operate at the low frequencies of 50Hz and 60Hz, transformers for signal transmission must operate over a wider range of frequencies — for example, the audio frequency range (20 Hz to 20kHz), or even higher for the radio frequency range.

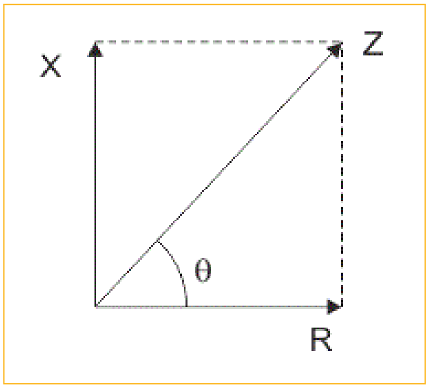
Using a transformer in your circuit that is not adequate for your frequency range can cause signal and power losses, as well as overheating. Careful manufacturing techniques and specialized materials are required to construct transformers that operate well over a wide range of frequencies.

As a rule, you should always consult the manufacturer before making a decision on an impedance matching transformer; this will help ensure you make the right choice for your application and achieve optimal performance.

CALCULATIONS, PROOFS, AND EXTRA INFORMATION

Knowing these proofs is not essential to selecting an impedance matching transformer, but it is useful for those who like to know how the mathematics work.

Impedance and Phase Differences



Impedance is a complex mathematical number, meaning it has both real and imaginary parts. It is a vector designated by the symbol Z and may be represented by Polar (Z/θ) or Cartesian coordinates (R+jX). It is made up by Resistance R (real) and Reactance jX (imaginary, with j denoting the imaginary factor).

Resistance is calculated from the familiar Ohm's Law relationship, R = V/I.

Reactance comes from the Capacitance (C) and Inductance (L) of the circuit, and the frequency of the applied voltage. It is denoted by the symbol X followed by C or L depending on the reactance it represents. Capacitance (C) is measured in Farads (F) and Inductance (L) is measured in Henry (H) — these values depend on the physical properties of the circuit's electrical components and will not be further covered here. Inductive Reactance is the positive section of the Reactance axis, while Capacitive Reactance is the negative section.

The equation for Inductive Reactance is given by (with f as the frequency in Hz): $X_L = 2\pi f L$

The equation for Capacitive Reactance is given by (with f as the frequency in Hz): $X_C = \dfrac{1}{2\pi f C}$

Total Reactance for a circuit is given by $X_L - X_C$

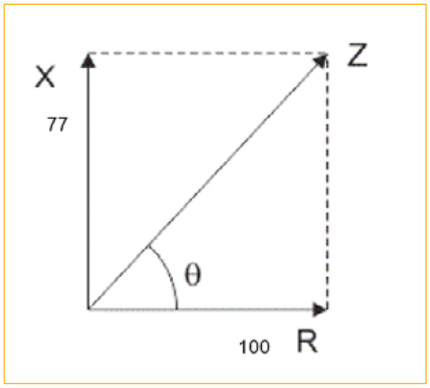
θ denotes the phase angle between voltage and current in the circuit. We can see that in a circuit with just a resistive component (DC circuits), θ = 0°, and the voltage and current are in phase.

A circuit with a positive total reactance (inductive circuit) and no resistance will have θ =90°, and the current is said to lag the voltage by 90°. A circuit with negative total reactance (capacitive circuit) and no resistance will have θ =-90°, and the current is said to lead the voltage by 90°.

In real-world AC applications, a circuit will have both reactive and resistive components and will be used to calculated the magnitude and phase angle of the impedance.

Example

A circuit with an inductance of 500mH, a capacitance of 100μF, and a resistance of 100Ω is connected to a 120V, 60Hz AC power supply.



$$X_L = 265/90^\circ\Omega$$

$$X_C = 188/-90^\circ\Omega$$

$$X = 77/90^\circ\Omega$$

The magnitude and phase angle of the impedance can be calculated by simple vector addition or trigonometry.

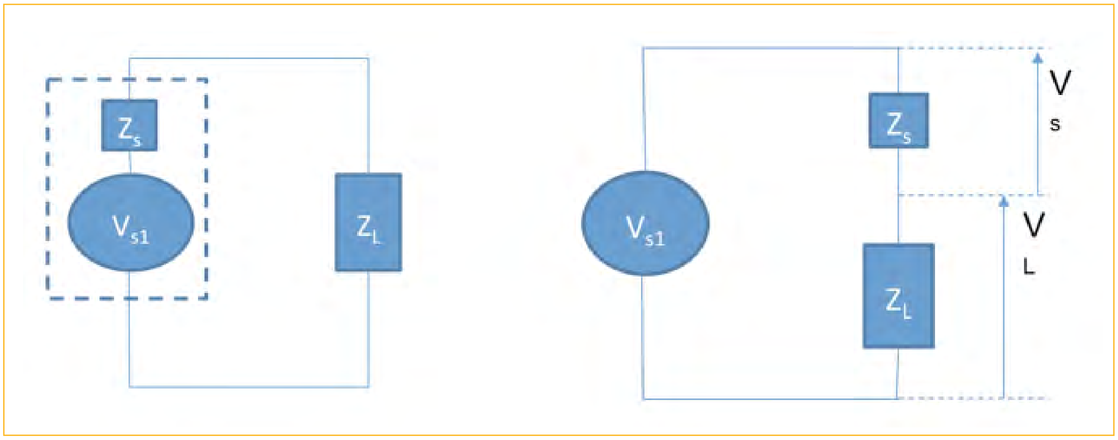
$$Z \text{ magnitude} = \sqrt{77^2 + 100^2}$$

$$\theta = \tan^{-1}(77/100)$$

This gives a total impedance Z of: $Z = 100 + j77\Omega$ $Z = 126/37.5^\circ\Omega$

Maximum Power Theorem

Maximum Power Theorem finds the optimum ratio of load impedance to source impedance for power transfer. The first circuit shows a power supply with an internal impedance connected to a load. This circuit can be redrawn to show how the applied voltage is divided between them, as shown:



For the redrawn circuit, the total current flowing is:

$$I = \frac{V_{s1}}{Z_S + Z_L}$$

Power transferred to the load is:

$$P_L = I^2 Z_L = \left(\frac{V_{s1}}{Z_S + Z_L}\right)^2 Z_L$$

In the above equation, Z_L is the value for which we are trying to find the optimum value. The condition for maximum power delivered to the load can be determined by differentiating load power with respect to the resistance and equating it to 0 (the peak of the power/resistance curve).

$$\frac{\partial P_L}{\partial Z_L} = V_{s1}^2 \left[\frac{(Z_S + Z_L)^2 - 2Z_L (Z_S + Z_L)^2}{(Z_S + Z_L)^4}\right] = 0$$

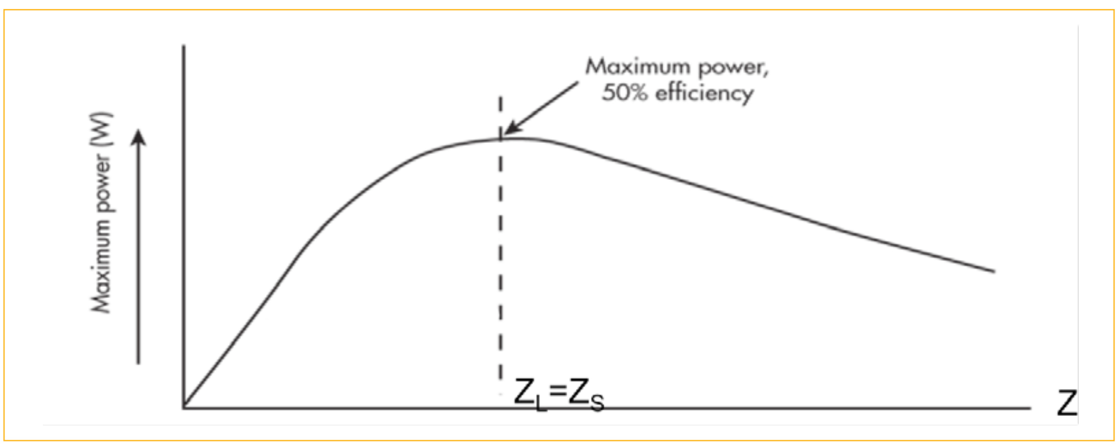
Solving...

$$(Z_S + Z_L)^2 = 2Z_L (Z_S + Z_L)^2$$

$$Z_S + Z_L = 2Z_L$$

$$Z_S = Z_L$$

This can be shown on the power/resistance graph as such:



Transformers

We know from Faraday's Law of Inductance that electromotive force ε (in this case voltage) is proportional to N number of turns in the winding and dϕ/dt the rate of change of magnetic flux.

$$\varepsilon = N \frac{d\phi}{dt}$$

Faraday's Law applies to both coils of a transformer, giving:

$$V_p = N_p \frac{d\phi}{dt} \qquad V_S = N_S \frac{d\phi}{dt}$$

As the rate of change of magnetic flux is the same across both coils, this gives:

$$\frac{V_p}{N_p} = \frac{V_S}{N_S} \qquad \frac{V_p}{V_S} = \frac{N_P}{N_S} = n$$

We know that power P in both the primary and secondary circuits is the same (ignoring small losses and assuming currents are the maximum rated currents of the transformer windings), so we can use:

$$P_P = I_P V_P = P_S = I_S V_S \qquad \frac{V_p}{V_S} = \frac{I_S}{I_P} = n$$

$$I_P V_P = I_S V_S$$

We can also use Ohm's Law to determine the impedance ratio of a transformer. To be clear, the impedances determined here are not the same as the self-impedances of the transformer windings (the measured impedance of an unloaded transformer coil). Transformer impedance ratings ony apply when the transformer is properly connected and appropriately loaded.

The impedance looking into each side of the transformer is given by:

$$V=IZ$$

From this, we can see the transformer Impedance Ratio, n is given by:

$$n = \frac{I_p Z_p}{I_S Z_S} = \frac{1 Z_p}{n Z_S} \qquad n^2 = \frac{Z_p}{Z_S}$$

Learn More

By matching load impedance, impedance matching transformers play a critical role in ensuring smooth operations in a range of industries and applications. Since the ratio of impedance in the source and the load affect how much power can be transferred from the source to the load, source impedance and load impedance must be equal in magnitude. Load impedance matching allows for maximum power transfer; the Maximum Power Theorem is used to determine the optimal ratio of load impedance to source impedance for power transfer.

To learn more about impedance matching transformers and for help in selecting the right type for your specific application, reach out to the team at Triad today.

ABOUT TRIAD MAGNETICS

EXPERIENCE

With more than 500 transformer manufacturers in the world, we realize you have a choice. Why choose Triad Magnetics? Having served the needs of our industry for more than half a century, we believe our experience makes the difference. If there is one point our experience has taught us, it is that we must remain flexible and adaptable to the changing needs of the market.

STANDARD OR CUSTOM

Over 1,000 part numbers mean you will probably find the component you need in our standard product line. If not, the creative thinkers of Triad Magnetics can offer powerful custom solutions. Whether it's Switchmode/High Frequency, Wall Plug-In, Power Transformers, Inductors or Audio Transformers, each product is backed up by the industry's most resourceful and organized magnetic supplier organization.

DESIGN INNOVATION

Triad Magnetics' pioneering design process promotes innovation. There are thousands of Triad Magnetics designs providing application solutions throughout the world, from data processing to telecommunications to power conversion. Our engineers are experienced in all packaging styles, even paper encapsulated units. Each engineer has direct ownership and manages projects from initial concept through production.

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